On a Question of Comfort and Negrepontis

Luis Recoder-Núñez

(Joint Work with W.W. Comfort and Ivan Gotchev)

Abstract

Here, α is an infinite cardinal; $\{X_i : i \in I\}$ is a set of spaces, and $X_J := \Pi_{i \in J} X_i$ when $\emptyset \neq J \subseteq I$; Y is dense in X_I, Z is a space and $f : Y \to Z$ is continuous. f is said to *depend on* J if $p, q \in Y$, $p_J = q_J \Rightarrow f(p) = f(q)$; in this case, $f_J : \pi_J[Y] \to Z$ is well-defined by the rule $f = f_J \circ \pi_J$.

In the book "Chain Conditions in Topology", Comfort and Negrepontis showed that if (a) $\pi_J[Y] = X_J$ for all $J \in [I]^{<\alpha}$ and (b) f depends on some $J \in [I]^{<\alpha}$, then f extends continuously over X_I. They asked whether, retaining (b), (a) may be replaced by (a$'$) $J \in [I]^{<\alpha} \Rightarrow$ every continuous $g : \pi_J[Y] \to Z$ extends continuously over X_J.

The present authors provide a theorem in the positive direction and study related questions.

Theorem. For fixed continuous $g : X_I \to Z$, $J \subseteq I$ and dense $Y \subseteq X_I$, TFAE:
(i) $f := g|Y$ depends on J and f_J is continuous.
(ii) $g|\pi_J^{-1}[\pi_J[Y]]$ depends on J.
(iii) g depends on J.

Theorem. Let f depend on J. Then f extends to continuous $\overline{f} : X_I \to Z$ that depends on J iff f_J is continuous and extends continuously over X_J.

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