CCSU

DEPARTMENT OF MATHEMATICAL SCIENCES

COLLOQUIUM

Friday, October 1
2:00 – 3:00 PM
Maria Sanford, Room 101

QUADRATIC INTEGERS: SOME PROPERTIES AND HISTORY

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Abstract: A high school algebra class teaches one how to add, subtract, multiply, and divide numerical expressions including square roots. For example, $(1 + \sqrt{3}) + (5 - 2\sqrt{3}) = (6 - \sqrt{3})$. However, such numbers have long played an important role in number theory, and there are still open questions about them today. The first half of this talk defines what quadratic integers are using some basic ideas from abstract algebra, which are illustrated by plenty of concrete examples. The second half of this talk gives some of the highlights of the history of quadratic integers focusing on the contributions by Fermat, Euler, Lagrange, Gauss, and Dedekind.

Here are some questions that will be answered in this talk.

(1) $644316469098589249480878067748476 87^2 + 611324130776250717917583810113577 84^2 = 888178419700125232338905334472656 25^2$. How is this fact related to $1 + 2\sqrt{(-1)}$?

(2) 1000000009 is both prime and congruent to 1 (mod 4). Euler proved that this implies 1000000009 is the sum of two squares. However, how can these two squares be explicitly computed?

(3) The following number has norm 1 as a quadratic integer: $152139002499 + 107578520350\sqrt{2}$. How does one find such examples?

(4) $x^2 + 2y^2$ has very different properties than $x^2 + 5y^2$ when viewed as a quadratic form. How did Gauss explain this distinction? What are the consequences of this distinction for quadratic integers?

(5) $6 = 2*3 = (1 + \sqrt{(-5)})(1 - \sqrt{(-5)})$ is a factorization of 6 into irreducibles in two different ways. How does Dedekind's theory of ideal factorization restore unique factorization, and what do these ideals look like when plotted on the complex plane?

For further information:

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