

CCSU  
DEPARTMENT OF MATHEMATICAL SCIENCES

# COLLOQUIUM

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2:00 – 3:00 PM  
Maria Sanford, Room 101

## WHATS IS AN $A_\infty$ -SPACE?

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### ABSTRACT

The associative law for the real numbers states that  $(ab)c=a(bc)$ , where  $a, b, c$  are real numbers. There is, of course, an associative law for addition, and in general, mathematicians frequently encounter binary operations defined on a set  $X$ ,

$$\mathbf{m}: X \times X \rightarrow X,$$

which also satisfies the associative law.

However, there are also situations in which the binary operation  $\mathbf{m}$  is not associative, but is very close to being associative. For example, if  $Y$  is a topological space, the based loop space of  $Y$  has a binary operation given by loop concatenation. But this operation is not strictly associative. In this case, we can measure the deviation from satisfying the associative relation in relation to a three-to-one operation. The binary (two-to-one) and three-to-one operations must satisfy a condition, which can be described in relation to a four-to-one operation. The process continues in this way to define infinitely many operations satisfying certain compatibilities. The based loop space together with all the  $n$ -to-one operations define an  $A(ssociative)_\infty$ -space.

In this talk, we will describe the above process and give a definition of an  $A_\infty$ -space. This will allow us to state the Recognition Theorem, which shows why these objects are of interest to homotopy theorists.

*For further information:*

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