CCSU
DEPARTMENT OF MATHEMATICAL SCIENCES

COLLOQUIUM

Friday, December 6
2:00 – 3:00 PM
Maria Sanford, Room 101

PRIME NUMBER DENSITY
IN THE INTERVAL (X,2X]

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<u>Abstract:</u> Assuming the validity of the Riemann Hypothesis (RH), it follows that

$$\lim_{x\to\infty}\left(\ln x - \frac{x}{\pi(x)}\right) \coloneqq k_\infty$$

exists and equals an interesting number. However, this limit result is valid <u>without</u> RH and hence $\frac{1}{-k_\infty + \ln x}$ is the optimal $\frac{\pi(x)}{x}$ approximator in the "Legendre" family $\frac{1}{C + \ln x}$. The limit

$$\lim_{x\to\infty}\left(2 - \frac{\pi(2x)}{\pi(x)}\right)\ln x$$

also exists, and hence again the limit value is an interesting number. A combination of the limit results yields the possibly suprising conclusion

$$\frac{1}{\ln 2x} < \frac{\pi(2x) - \pi(x)}{x} < \frac{1}{\ln x},$$

valid for x sufficiently large.