CCSU
DEPARTMENT OF MATHEMATICAL SCIENCES

VIRTUAL COLLOQUIUM

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3:00 – 4:00 PM
https://ccsu.webex.com/meet/gotchev

FIBONACCI NUMBERS AND QUADRATIC INTEGERS

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Abstract: In these challenging times, there is nothing better to lift one's spirits than the Fibonacci numbers, the famous sequence defined by $F_n = F_{n-1} + F_{n-2}$, and $F_1 = F_2 = 1$. We will begin with a derivation of Binet's formula using hierarchical regression, a technique from statistics. In doing so we will see there is an intimate connection between 2nd order, linear recurrence relationships, $f_n = af_{n-1} + bf_{n-2}$, and the monic quadratic, $x^2 - ax - b$, where a and b are integers. For example, the Fibonacci numbers are associated with $x^2 - x - 1$, which can be used to derive Binet's formula. However, fans of algebraic number theory know that solving $x^2 - ax - b = 0$ defines the quadratic integers, and the rest of this talk shows some results arising from this link. For instance, for φ, the Golden Ratio, $Z[\varphi] = \{a + b\varphi : a, b \in Z\}$ is the ring of algebraic integers of $Q[\sqrt{5}]$. The group of units of $Z[\varphi]$ are isomorphic to $Z_2 \times Z$, which is an example of Dirichlet's Unit Theorem. In conclusion, if you enjoy things like φ, rational approximation, and Pell's equation, this talk is for you.

For further information:
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