CCSU
DEPARTMENT OF MATHEMATICAL SCIENCES

COLLOQUIUM

Friday, September 9
3:00 – 4:00 PM
Maria Sanford, Room 101

PROPERTIES OF OCTONION PRODUCTS

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Abstract: The four division algebras R, C, H and O (the reals, the complex numbers, the quaternions and the octonions) are described as follows in "The Octonions" by John Baez:

> "The real numbers are the dependable breadwinner of the family, the complete ordered field we all rely on. The complex numbers are a slightly flashier but still respectable younger brother: not ordered, but algebraically complete. The quaternions, being non-commutative, are the eccentric cousin who is shunned at important family gatherings. But the octonions are the crazy old uncle nobody lets out of the attic: they are nonassociative."

We will review some of the peculiar properties of octonion multiplication including the "alternative" property of the octonion "associator", $(xy)z - x(yz)$: In general, it is non-zero, but it changes sign whenever you switch two of its arguments. We also define and discuss the 7-dimensional cross product of a pair of pure imaginary octonions and prove the following formula for the "anti-associator":

$$(\mathbf{x} \times \mathbf{y}) \times \mathbf{z} + \mathbf{x} \times (\mathbf{y} \times \mathbf{z}) = 2(\mathbf{x} \cdot \mathbf{z})\,\mathbf{y} - (\mathbf{x} \cdot \mathbf{y})\,\mathbf{z} - (\mathbf{y} \cdot \mathbf{z})\,\mathbf{x}$$

From this follows the alternative property of the full octonion associator.

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