CCSU

DEPARTMENT OF MATHEMATICAL SCIENCES

COLLOQUIUM

Friday, February 6
2:00 – 3:00 PM
Maria Sanford, Room 101

EXAMPLES OF SOLVING SOLVABLE POLYNOMIALS IN RADICALS

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Abstract: The quintic $x^5 + 15x + 12 = 0$ can be solved using radicals. How can one know this? and how can the solutions (the real one is given below) be explicitly written out? This talk will answer these questions and more. We begin with quadratics and derive the well-known $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ in several ways. However, this programmed on a computer can give numerically inaccurate answers (when is this true?) Rationalizing the numerator produces $\frac{2c}{-b \mp \sqrt{b^2 - 4ac}}$, which is the first step to correcting this problem (why does this help?) Being warmed up, we next derive a solution (or two) for both the cubic and quartic, giving explicit examples using Mathematica. At this point, we turn to Lagrange's unified approach to solving these three types of polynomials, which he published in 1770. His hope was to extend his techniques to the quintic, and although he failed, his ideas are now part of Galois theory. Using group theoretic ideas, we find out how to identify whether or not a specific quintic is solvable, and when it is, how to write down its roots using radicals. Finally, we finish by with a step-by-step derivation of Gauss' solution to $x^{17} - 1 = 0$, by which he proved a regular 17-gon is constructible using only a compass and unmarked straightedge. This talk is entirely written in Mathematica, so be ready for plenty of explicit computations, almost all of which were originally done by hand before 1900.

$$-\frac{1}{5^{4/5}} 3^{1/5} \left(\left(5-\sqrt{10}\right)^{2/5} \left(-5+2\sqrt{10}\right)^{1/5} + \left(5-\sqrt{10}\right)^{1/5} \left(5+2\sqrt{10}\right)^{2/5} + \left(5+\sqrt{10}\right)^{1/5} \left(-\left(15\left(3+\sqrt{10}\right)\right)^{1/5} + \left(-5+2\sqrt{10}\right)^{2/5}\right)\right)$$

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