CCSU
DEPARTMENT OF MATHEMATICAL SCIENCES

COLLOQUIUM

Wednesday, April 4

9:45 – 10:40 in LD 207

GEOMETRIC INTERPRETATION OF THE TWO DIMENSIONAL POISSON KERNEL AND ITS APPLICATIONS

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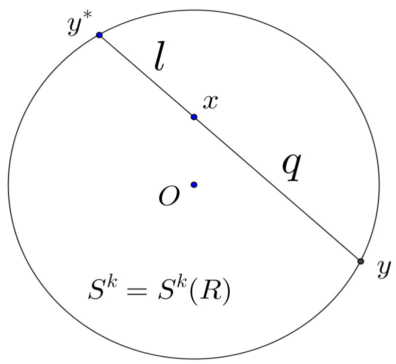
Herman Schwarz, while studying complex analysis at the end of 19-th century, introduced the geometric interpretation for the two-dimensional Poisson Kernel (ω). In this presentation we shall see that the geometric interpretation considered in a multidimensional space and combined with the planar geometry can serve as a useful tool to obtain a number of results related to Euclidean and Hyperbolic geometry.

In particular, as the main result, we shall see that this 2-D Poisson kernel satisfies the following integral equivalence for all real numbers α and β as well as, with some restrictions, for complex numbers α and β.



$$\int_{S^k} \omega^{\alpha} dS_y = \int_{S^k} \omega^{\beta} dS_y \iff \alpha + \beta = k$$

$$\text{where } \omega(x, y) = l/q = \frac{|x - y^*|}{|x - y|}$$

is the two-dimensional Poisson Kernel, usually used to solve Dirichlet Problem in a planar disk or in its exterior and k is the dimension of the sphere S^k(R) in a (k+1)-dimensional space of radius R centered at O.
We assume |x|≠R=|y|.

his integral equivalence applied for complex numbers α and β leads to One Radius Theorem saying that two radial eigenfunctions of a hyperbolic Laplacian assuming the same value at the origin cannot assume any other common value within some interval [0,p], where the length of this interval depends only on the location of the eigenvalues on the complex plane and does not depend on the distance between them.

The technique used in obtaining that integral equivalence as well as the equivalence in itself let us derive the following results:

In Euclidean Space: 1) A new algebraic way to compute certain integrals arising in electrostatics; 2) A new derivation for a solution of the classical Dirichlet problem in (k+1)-dimensional ball of radius R or in its exterior. This derivation does not involve Green's identity or Green's function; 3) Some non-trivial inequalities; 4) A sufficient condition for a function depending only on distance to be harmonic.

In Hyperbolic Space: 1) Explicit computation of all eigenvalues for the Dirichlet Eigenvalue Problem in 3-dimensional hyperbolic ball. The corresponding radial eigenfunctions also can be computed explicitly; 2) The lower and upper bounds for the minimal eigenvalue in a Dirichlet eigenvalue problem; 3) Analysis of the behavior of radial eigenfunctions at infinity.