CCSU

DEPARTMENT OF MATHEMATICAL SCIENCES

COLLOQUIUM

Friday, March 22
2:00 – 3:00 PM
Maria Sanford, Room 101

AROUND WEYL'S UNIFORM DISTRIBUTION MODULO ONE

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Abstract: Let $\mathbf{m} = (m_n)$ be a one-to-one sequence of integers. According Weyl's theorem the sequence $(m_n\alpha)$ is uniformly distributed modulo one for almost all $\alpha \in \mathbb{R}$. More precisely, passing to the circle group $\mathbb{T} = \mathbb{R}/\mathbb{Z}$, the set

$$W_{\mathbf{m}} = \{\alpha \in \mathbb{T}: (m_n\alpha) \text{ is uniformly distributed in } \mathbb{T}\}$$

is a Borel set of full measure in $\mathbb{T}$. The talk is dedicated to the "complement" (to a certain extent) of Weyl's set $W_{\mathbf{m}}$, namely the set

$$C_{\mathbf{m}} = \{\alpha \in \mathbb{T}: \lim_n m_n\,\alpha = 0 \text{ in } \mathbb{T}\}. \quad (*)$$

The sets of the form (*) have been studied in descriptive set theory, number theory, topology and analysis (in connection to trigonometric series). Even if $C_{\mathbf{m}}$ is not exactly the complement of $W_{\mathbf{m}}$, it shares many of its properties (e.g., it is Borel set of measure zero in $\mathbb{T}$) and it is a *subgroup* of $\mathbb{T}$ (unlike $W_{\mathbf{m}}$).

The talk will discuss many examples as well as some general results (e.g., for the Fibonacci sequence **m** the subgroup $C_{\mathbf{m}}$ of $\mathbb{T}$ is cyclic, generated by the Golden Ratio $\alpha = \frac{1+\sqrt{5}}{2}$). One can prove that every countable subgroup of $\mathbb{T}$ has the form (*) for an appropriate sequence $(m_n\alpha)$, yet:

- many F_σ-subgroups of $\mathbb{T}$ may fail to have this form;
- many subgroups of the form (*) may fail to be F_σ-sets (e.g., for $m_n = n!$, or $m_n = 2^{2^n}$).

For further information:
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