CCSU
DEPARTMENT OF MATHEMATICAL SCIENCES
COLLOQUIUM

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2:00 – 3:00 PM
Maria Sanford, Room 101

A CHARACTERIZATION OF QUADRIC CONSTANT SCALAR CURVATURE HYPERSURFACES OF SPHERES

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Abstract: Let $S^4=\{x \in R^5 : |x|=1\}$ be the unit 4-dimensional sphere. Easy examples to define in this manifold are $M_1(c)=\{(x_1, x_2, x_3, x_4, x_5) \in S^4: x_5 = c\}$ where $c \in (-1,1)$ and $M_2(r)=\{(x_1, x_2, x_3, x_4, x_5) \in S^4: x_4^2+ x_5^2= r^2\}$ where $r \in (0,1)$. It is not difficult to see that $M_1(c)$ is a 3-dimensional sphere with radius $(1-c^2)^{1/2}$ and $M_2(r)$ is the Cartesian product of a circle of radius r and a 2-dimensional sphere of radius $(1-r^2)^{1/2}$. Up to rigid motions, a quadratic hypersurface in S^4 is a hypersurface that can be written as $M_2(r)$ for some r. Due to its large group of symmetries, both families of hypersurfaces $M_1(c)$ and $M_2(r)$ have constant scalar curvature.

Let us consider a hypersurface $M \subset S^4$. For every point $x \in M$, the tangent space T_xM is a three dimensional space in R^5 which can be shown is perpendicular to the vector x. Therefore, the vector space

$$W(x)=\{v+\lambda x: v \in T_xM , \lambda \in R\}$$

is a 4-dimensional space of R^5. By basic linear algebra we know that up to a sign, there exists a unique vector N(x) in R^5 that is perpendicular to the vector space W(x). When M is orientable, we can choose a continuous function $N:M\rightarrow R^5$, such that N(x) has norm 1 and N(x) is perpendicular to W(x). This map is called a Gauss map. In this talk we will present the following result

Theorem (Perdomo-Wei, to appear in the Journal of Geometric Analysis): Let $M \subset S^4$ is a complete (topologically complete space) orientable hypersurface with constant scalar curvature and let $N:M\rightarrow R^5$ be its Gauss map. If for some fixed vector $v \in R^5$ and some real number λ, we have

$$< N(x),v > = \lambda <x , v> \text{ for all } x \in M$$

then, up to a rigid motion, M is equal to either $M_1(c)$ for some c or $M_2(r)$ for some r.
(Here $<u,w>=u_1 w_1 + u_2 w_2 + u_3 w_3 + u_4 w_4 + u_5 w_1$ is the standard inner product in R^5.)

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