CCSU

DEPARTMENT OF MATHEMATICAL SCIENCES

COLLOQUIUM

Friday, February 8

2:00 – 3:00 PM

Maria Sanford, Room 101

ONE RADIUS THEOREM FOR TWO RADIAL DIRICHLET EIGENFUNCTIONS IN A HYPERBOLIC SPACE

SERGEI ARTAMOSHIN

CENTRAL CONNECTICUT STATE UNIVERSITY

ABSTRACT Consider the set all radial eigenfunctions of the Hyperbolic Laplacian assuming the value one at the origin, i.e., the set of all solutions for the following system

$$\begin{cases} \varphi''(r) + \frac{k}{\rho}\coth\left(\frac{r}{\rho}\right)\varphi'(r) + \lambda\varphi(r) = 0, \ \ \lambda \in \mathbb{C} \\ \varphi(0) = 1, \end{cases}$$

written in the geodesic polar coordinates of the hyperbolic space of constant sectional curvature $\kappa = -1/\rho^2$. It is known that for every $\lambda \in \mathbb{C}$ there exists a unique solution $\varphi_\lambda(r)$ such that $\varphi_\lambda(0) = 1$. If we choose two radial eigenfunctions $\varphi_\mu(r)$ and $\varphi_\nu(r)$ such that $\varphi_\mu(0) = \varphi_\nu(0) = 1$, then, how many values of $r > 0$ such that $\varphi_\mu(r) = \varphi_\nu(r)$ are sufficient to ensure that $\mu = \nu$ and $\varphi_\mu(r) \equiv \varphi_\nu(r)$?

We shall see that we need only one value of r, if the value is small enough. In other words, if $\mu \neq \nu$, then there exists an interval $(0, p(\mu,\nu)]$, such that $\varphi_\mu(r) \neq \varphi_\nu(r)$ for all $r \in (0, p(\mu,\nu)]$.

In particular, we prove, that if $\mu \neq \nu$ are real, and $\mu, \nu \leq k^2/4$ then $\varphi_\mu(r) \neq \varphi_\nu(r)$ for all $r \in (0, \infty)$. Or, vise-versa, if $\mu, \nu \leq k^2/4$ and $\varphi_\mu(r) = \varphi_\nu(r)$ just for one arbitrary $r > 0$, then $\mu = \nu$ and $\varphi_\mu(r) \equiv \varphi_\nu(r)$.

Remark. The theorem will be obtained by analysis of the following implication.

$$\int_{S^k} \omega^\alpha dS_y = \int_{S^k} \omega^\alpha dS_y \ => \ \alpha + \beta = k,$$

where S^k is the k-dimensional sphere of radius R centered at the origin; $x \in R^{k+1} \backslash S^k$; α, β are two different complex numbers and $\omega = \omega(x,y)$ is the two-dimensional Poisson kernel used to solve Dirichlet Problem in a planar disk.

For further information:

gotchevi@ccsu.edu 860-832-2839

http://www.math.ccsu.edu/gotchev/colloquium/