CCSU

DEPARTMENT OF MATHEMATICAL SCIENCES

COLLOQUIUM

Friday, April 26

2:00 – 3:00 PM

Maria Sanford, Room 101

ASSYMPTOTIC BEHAVIOR OF RADIAL EIGENFUNCTIONS OF THE HYPERBOLIC LAPLACIAN FOR NEGATIVELY LARGE EIGENVALUES

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ABSTRACT A radial eigenfunction of the Hyperbolic Laplacian is a solution of the following system

$$\begin{cases} \varphi''(r) + \frac{k}{\rho}\coth\left(\frac{r}{\rho}\right)\varphi'(r) + \lambda\varphi(r) = 0, \ \ \lambda \in \mathbb{C} \\ \varphi(0) = 1, \end{cases}$$

written in the geodesic polar coordinates of the hyperbolic space of constant sectional curvature $\kappa = -1/\rho^2$. It is known that for every $\lambda \in \mathbb{C}$ there exists a unique solution $\varphi_\lambda(r)$. Our goal is to investigate the behavior of $\varphi(r)$ as $\lambda \longrightarrow -\infty$.

The presentation will be accessible for calculus students. First, we obtain the integral form of $\varphi_\lambda(r)$ and then, introduce Laplace's Method as the basic tool to investigate the asymptotic behavior of integrals depending on a parameter. We shall see some elementary examples related to Laplace's Method and then apply the Laplace's theorem to obtain the leading term of $\varphi(r)$ as $\lambda \longrightarrow -\infty$.

As an application of the asymptotic behavior combined with Liouville's Theorem, we may conclude that the implication given below cannot be true for all complex numbers $\alpha \neq \beta$ satisfying the integral identity.

$$\int_{S^k} \omega^\alpha dS_y = \int_{S^k} \omega^\beta dS_y \ \Rightarrow \ \alpha + \beta = k,$$

where S^k is the k-dimensional sphere of radius R centered at the origin; $x \in R^{k+1}\backslash S^k$; α, β are two different complex numbers and $\omega = \omega(x, y)$ is the two-dimensional Poisson kernel used to solve Dirichlet Problem in a planar disk.

For further information:

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