## CCSU department of mathematical sciences COLLOQUIUM

Friday, February 4 3:00 – 4:00 PM Maria Sanford, Room 101

## APPLYING BINARY RELATIONS ROGER BILISOLY

## **CENTRAL CONNECTICUT STATE UNIVERSITY**

**Abstract:** A relation between elements of two sets, *X* and *Y*, shows how elements relate. For example, let  $X = \{0,1,2,3,4\}$  and Y = X. Then 'less than,' 'greater than,' and 'equals' are all relations. In general, a relation is merely any subset of the Cartesian product,  $X \times Y$ . This definition will be explained and illustrated with many examples with the help of Python and R. Next, using graph theory, and the Python package NetworkX, we completely solve the word game, Doublets. This was invented by the mathematician Charles Dodgson, better known as Lewis Carroll. The goal is to change one word to another (of the same length) by changing only one letter at a time, and every intermediate string must also be a word. For instance, Carroll published this challenge in *Vanity Fair*: change 'head' to 'tail.' His solution was head  $\rightarrow$  heal  $\rightarrow$  teal  $\rightarrow$  tell  $\rightarrow$  tall  $\rightarrow$  tail, but there is a shorter one – can you find it? To end the talk, we consider prime-number doublets. For primes of a given length, a graph is induced by the binary relation  $p_1 \sim p_2$ , which occurs when these two primes differ by exactly one digit (such as 1000037 and 1000039, which differ only in the last digit.) For 2-, 3-, 4-, and 5-digit primes, this graph is connected: see the example of 2-digit primes below. The talk ends with a statement of a theorem **proved** (not merely conjectured) by Terence Tao, which shows that connected n-digit-prime doublet graphs are the exception, not the rule.



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