TAKE HOME EXAM 2

Solve the following problems showing all your work for full credit.

1.   Convert the angle θ
   a) (2 pts.) $\theta = 750°$ to radian measure in terms of π ;

   b) (2 pts.) $\theta = -\dfrac{7\pi}{9}$ to degree measure.

2. (4 pts.) Find the distance along an arc on the surface of the earth that subtends a central angle of 1minute (1minute $= \dfrac{1}{60}$ degree). This distance is called a *nautical mile* . (The radius of the earth is 3960 mi.)

3. (4 pts.) The radius of the Earth is approximately 6,400 kilometers. If a central angle of $14°$, with vertex at the center of the Earth, intersects the surface of the Earth in town A and town B, what is the distance along the Earth's surface between A and B? Round to the nearest kilometer.

4. In a circle with radius 55 in, an arc is intercepted by a central angle $\theta = 127°$.
   a) (3 pts.) Find the arc length. (Round to two decimal places.)

   b) (3 pts.) Find the area of the circular sector. (Round to two decimal places.)

5. (4 pts.) A sprinkler has a 30-foot spray and covers an angle of $75°$. What is the area that the sprinkler waters?

6. (4 pts.) Find the angular and linear speed of a point that moves with constant speed in a circular motion if the point travels along a circle with radius $r = 5$ km, arc length $s = 2.3$ km in time $t = 4$ minutes.

7. (4 pts.) A car owner decides to upgrade from tires with a diameter of 24.8 inches to tires with a diameter of 27.0 inches. If she does not update the onboard computer, how fast will she actually be traveling when the speedometer reads 70 mph?

8. (4 pts.) How fast is a bicyclist traveling in miles per hour if his tires are 22 inches in diameter and his angular speed is 5π radians per second?

9. (4 pts.) Calculate the expression $\dfrac{2\sin\left(-\dfrac{\pi}{3}\right) - 4\cos\left(-\dfrac{5\pi}{3}\right)}{\cos\left(-\dfrac{5\pi}{4}\right)}$.

10. Find all the exact values of θ that are solutions of the given equation in the indicated interval.

    a) (3 pts.) $\cos\theta = -1$, $0 \le \theta \le 4\pi$;

    b) (3 pts.) $\sec\theta = -\sqrt{2}$, $0 \le \theta \le 2\pi$;

    c) (3 pts.) $\sin\theta = 0$, $0 \le \theta \le 4\pi$;

    d) (3 pts.) $\tan\theta = 1$, $0 \le \theta \le 2\pi$.

11. State the amplitude and period of each function:

    a) (3 pts.) $y = \dfrac{3}{2}\sin(\dfrac{2}{3}x)$;

    b) (3 pts.) $y = -\dfrac{1}{3}\sin(\dfrac{1}{4}x)$;

    c) (3 pts.) $y = -3\cos(\pi x)$.

12. Graph the given function over one period. Determine the domain and range of the function based on this one period.

    a) (4 pts.) $y = -3\sin(2x)$;

b) (4 pts.) $y = 2\cos(\dfrac{1}{2}x)$.

13. Consider the function $y = -\dfrac{1}{2}\cos(2x - \dfrac{\pi}{2}) + 2$.

a) (1 pt.) Find the amplitude;

b) (1 pt.) Find the period;

c) (1 pt.) Find the phase shift;

d) (5 pts.) Sketch a graph of the function (without using a calculator). Show the graphs of all transformations starting with $y = \cos x$.

14. Determine the period and phase shift (if there is one) for each function:

a) (3 pts.) $y = 2\tan(2x - \dfrac{\pi}{3})$;

b) (3 pts.) $y = \sec(2x + \dfrac{\pi}{2})$.

15. Simplify:

a) (5 pts.) $\dfrac{1+\sin x}{\cos x} + \dfrac{\cos(-x)}{1+\sin x} =$

b) (4 pts.) $\dfrac{\cos^2 x + \cos x - 12}{\cos x + 4} =$

16. Verify each trigonometric identity:

a) (5 pts.) $\dfrac{1}{1-\cos x} + \dfrac{1}{1+\cos x} = 2\csc^2 x$

b) (5 pts.) $\dfrac{\sec x + \tan x}{\csc x + 1} = \tan x$