MATH 152 **NAME:**
CALCULUS I **ID#:**

TAKE-HOME EXAM 1

Solve the following problems showing all your work for full credit.

1. Find the domain of the functions and express it in interval notation:

a) (2 pt.) $f(x) = \frac{x+2}{x^2-9}$;

b) (2 pt.) $\sqrt{4-x^2}$;

c) (2 pt.) $f(x) = \frac{x-3}{(x-2)\sqrt{x+1}}$.

2. Which of the following functions are odd? Even? Neither even nor odd?

a) (1 pt.) $f(x) = \frac{5x^5}{3x^2-1}$;

b) (1 pt.) $g(x) = |x^3|$;

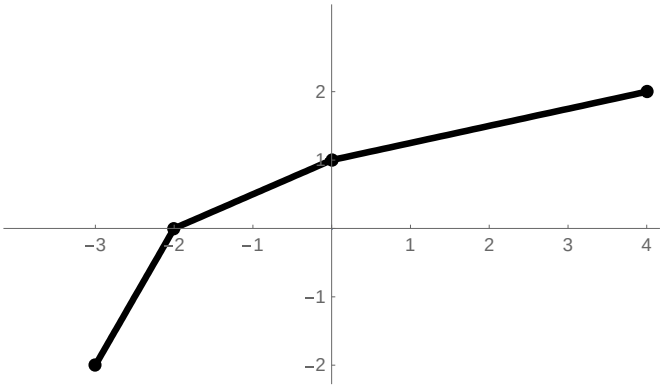
c) (1 pt.) $h(x) = 3x^4 + 5\cos x$.

3. A function f is defined by $f(x) = \begin{cases} x^2 - 1 & , \quad x < 0 \\ 1 & , \quad 0 < x \leq 1. \\ 3 - x & , \quad 1 < x \end{cases}$

a) (3 pts.) Evaluate $f(-3)$, $f(1)$, and $f(6)$;

b) (3 pts.) Make a hand-drawn graph of $f(x)$.

4. The graph of a function $y = f(x)$ is shown below. No formula for $f(x)$ is given.



a) (2 pts.) Find the domain and range of $f(x)$.

b) (3 pts.) Sketch the graph of the inverse function $f^{-1}(x)$;

c) (3 pts.) Sketch the graph of the function $g(x) = 2f(x+1) - 1$.

5. (2 pts.) Let $f(x) = x^2 - 2x$ and $g(x) = x + 2$. Find $f \circ g$ and $g \circ f$.

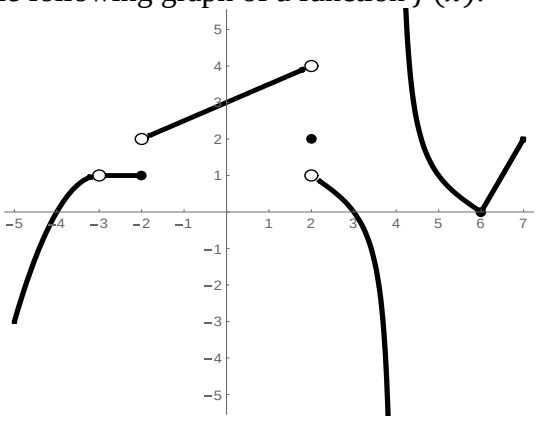
6. Calculate each of the following without using a calculator.

a) (1 pt.) $\sin 330°$

b) (1 pt.) $\cos \frac{8\pi}{3}$

c) (1 pt.) $\tan \frac{4\pi}{3}$

7. (13 pts.) Consider the following graph of a function $f(x)$:



Find the limit, if it exists.

a) $\lim_{x\to -4} f(x) =$ b) $\lim_{x\to -3^-} f(x) =$ c) $\lim_{x\to -3^+} f(x) =$

d) $\lim_{x\to -3} f(x) =$ e) Find $f(-3) =$ f) $\lim_{x\to 4^-} f(x) =$

g) $\lim_{x\to 2^-} f(x) =$ h) $\lim_{x\to 2^+} f(x) =$ i) $\lim_{x\to 2} f(x) =$

j) Find $f(2) =$ k) Is $f(x)$ continuous at -3?

l) Is $f(x)$ continuous at 2? m) Is $f(x)$ continuous at 6?

8. Find the limit if it exists:

a) (1 pt.) $\lim_{x\to 1} \frac{-2x^2+3}{5x-1} =$

b) (1 pt.) $\lim_{x\to 3} \cos\frac{\pi x}{12} =$

c) (2 pts.) $\lim_{x\to -5} \frac{x^2-25}{x+5} =$

d) (2 pts.) $\lim_{x\to 3} \frac{x^2-6x+9}{x-3} =$

e) (3 pts.) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} =$

f) (3 pts.) $\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}} =$

g) (3 pts.) $\lim_{x\to 0} \frac{1-(\cos 3x)^2}{(\tan 5x)^2} =$

h) (2 pts.) $\lim_{\theta \to 0} \frac{\cos(\pi-\theta)\tan\theta}{\theta} =$

i) (1 pt.) $\lim_{x \to 3^-} \frac{x^2-3x+9}{x-3} =$

j) (1 pt.) $\lim_{x \to 1} \frac{-2x+5}{x^2-2x+1} =$

k) (1 pt.) $\lim_{x \to -1^+} \frac{-x^2-5x+11}{(x-4)(x+1)} =$

l) (1 pt.) $\lim_{x \to 1^-} \frac{x^2-3x+1}{x-1} =$

m) (1 pt.) $\lim_{x \to 3^+} \frac{x^2-6x+7}{(x-3)(x-1)} =$

9. Find the limit L. Then use the $\varepsilon - \delta$ definition to prove that the limit is L.

a) (7 pts.) $\lim_{x \to -2} (3x + 2) =$

b) (7 pts.) $\lim_{x \to 2} (x^2 - 5x) =$

10. (4 pts.) Explain why the function $f(x) = x^3 + 3x - 2$ has a zero in the interval [0, 1].

11. (6 pts.) Find all vertical asymptotes of the functions:

a) $g(x) = \frac{3x^2}{x^2-4}$;

b) $h(x) = \frac{x^2}{x-2}$

12. (3 pts.) Find the constant a such that the function $f(x) = \begin{cases} x^2 - 1 & , \quad x \leq 3 \\ 4x + a & , \quad x > 3 \end{cases}$ is continuous on the entire real line.

13. (4 pts.) Sketch the graph of a function that has domain [0,4] and is continuous on [0,2) and [2,4] but is not continuous on [0,4].

14. (2 pts.) At what points, if any, is the function $f(x)$ undefined? Discontinuous? If the function is discontinuous at some point(s), justify your answer?

$$f(x) = \begin{cases} 3x^2 - 12 & , \quad x \leq -2 \\ \sqrt{x+2} & , \quad -2 < x \leq 1 \\ -x + 3 & , \quad 1 < x < 2 \\ 1 & , \quad 2 < x \end{cases}$$

15. (4 pts.) The function $f(x) = \frac{x^4-5x^2+4}{x+2}$ is not defined at a certain point. How should it be defined to make it continuous at that point?