MATH 152
NAME:
CALCULUS I
ID#:

TAKE-HOME EXAM 3

Solve the following problems showing all your work for full credit.

1. (5 pts.) The formula for the volume of a cone is $V = \dfrac{1}{3}\pi r^2 h$. Find the rate of change of the volume when $r = 6$ inches if $h = 3r$ and $\dfrac{dr}{dt}$ is 2 inches per minute.

2. (5 pts.) Find the linearization $L(x)$ of $f(x) = \sqrt[3]{x}$ at $x = -8$ and use it to approximate $\sqrt[3]{-8.1}$.

3. (5 pts.) Show that the function $f(x) = x^3 + \dfrac{4}{x^2} + 7$ has exactly one zero in the interval $(-\infty, 0)$.

4. (5 pts.) Identify the critical points and find the maximum value and minimum value of the function $f(x) = x^3 - 12x$ on the closed interval $[-4,4]$.

5. (10 pts.) Find the open intervals on which the function $f(x) = x^4 - 4x^3 + 4x^2$ is increasing or decreasing and locate all relative extrema. Also find the open intervals on which the graph of $f(x)$ is concave upward or concave downward, and points of inflection.

6. (5 pts.) Sketch the graph of a function f having the following characteristics: $f(0) = f(2) = 0$, $f(1) = -3$, $f'(x) < 0$ if $x < 1$, $f'(1) = 0$, $f'(x) > 0$ if $x > 1$, and $f''(x) > 0$ for all x.

7. (10 pts.) Find the area of the largest isosceles triangle that can be inscribed in a circle of radius 4.

8. (20 pts.) Analyze and sketch a graph of the function $f(x) = \dfrac{x^3}{x^2 - 4}$. Label any intercepts, relative extrema, points of inflection, and any horizontal, vertical, and oblique asymptotes.

9. (5 pts.) Evaluate the limit $\displaystyle\lim_{x\to 1} \dfrac{\arctan x - \left(\dfrac{\pi}{4}\right)}{x-1}$.

10. (5 pts.) Evaluate the limit $\displaystyle\lim_{x\to 2^-} \dfrac{\sqrt{4-x^2}}{x-2}$.

11. (5 pts.) Evaluate the limit $\displaystyle\lim_{x\to\infty} \dfrac{2x^3}{\ln x}$.

12. (5 pts.) Evaluate the limit $\displaystyle\lim_{x\to\infty}\left(\sqrt{x^2+2x-1}-x\right)$.

13. (5 pts.) Evaluate the limit $\displaystyle\lim_{x\to 4^+}\left[3(x-4)\right]^{x-4}$.

14. (5 pts.) Evaluate the limit $\displaystyle\lim_{x\to\infty} \dfrac{e^{2x}}{x^2}$.

15. (5 pts.) Use Newton's method to estimate the one real solution of $x^3+3x+1=0$. Start with $x_0=0$ and then find x_2.