

TAKE-HOME EXAM 4

Solve the following problems showing all your work for full credit.

1. (12 pts.) Find the antiderivative $F(x) + C$ for each of the following.

a) $f(x) = 3x^{\frac{2}{3}}$

b) $f(x) = 3x^2 + \sqrt{x}$

c) $f(x) = x^2(x^3 + 5x^2 - 3x + \sqrt{3})$

d) $f(x) = \frac{\sqrt{2x} + x^2}{x^4}$

2. (21 pts.) Evaluate the integral.

a) $\int (e^{3x} + 4^x) dx$

b) $\int \frac{s(s+1)^2}{\sqrt{s}} ds$

c) $\int (t^2 - 2 \cos t) dt$

d) $\int \frac{x^2 + 5x + 6}{x + 3} dx$

e) $\int (\pi x^3 + 1)^4 3\pi x^2 dx$

f) $\int (5x^2 + 1) \sqrt{5x^3 + 3x - 2} dx$

g) $\int \frac{3y}{\sqrt{2y^2 + 5}} dy$

3. (4 pts.) A balloon, rising vertically with a velocity of 16 feet per second, releases a sandbag at the instant it is 64 feet above the ground.

a) How many seconds after its release will the bag strike the ground?

b) At what velocity will it hit the ground?

4. (6 pts.) Use the properties of summation to evaluate the sum $\sum_{i=1}^{10} (i^3 - 2i^2 + 5i - 1)$.

5. (6 pts.) Find a formula for the sum of n terms. Use the formula to find the limit as $n \rightarrow \infty$.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(1 + \frac{2i}{n} \right)^3 \left(\frac{2}{n} \right)$$

6. (6 pts.) Use the limit process to find the area of the region between the graph of the function $y = x^2 - x^3$ and the x -axis over the interval $[0,1]$. Sketch the region.

7. (3 pts.) Express the limit $\lim_{n \rightarrow \infty} \sum_{i=1}^n 6c_i(4 - c_i)^2 \Delta x$ as a definite integral on the interval $[0,4]$, where c_i is any point in the i -th subinterval.

8. (3 pts.) Sketch the region whose area is given by the definite integral $\int_3^5 (8-x)dx$.

9. (3 pts.) Given $\int_0^3 f(x)dx = 4$ and $\int_8^0 f(x)dx = -10$, find $\int_3^8 f(x)dx$.

10. (9 pts.) Find $F'(x)$.

a) $F(x) = \int_0^x (3t^3 + \sqrt{t})dt$

b) $F(x) = \int_1^{x^2+x} \sqrt{2z + \sin z} dz$

c) $F(x) = \int_{x^2}^{\sin x} \frac{t^2}{t^2+1} dt$

11. (3 pts.) Find the average value of the function $f(x) = 3x^2 - 3$ over the interval $[-1,3]$.

12. (4 pts.) Find the area of the region bounded by the graphs of the functions $y = x^2$ and $y = \sqrt[3]{x}$.

13. (20 pts.) Evaluate the definite integral.

a) $\int_1^3 (3x^2 + 5x - 4) dx$

b) $\int_1^4 (3 - |x - 3|) dx$

c) $\int_{-\pi/2}^{\pi/2} (2t + \cos t) dt$

d) $\int_0^1 \cos(4x - 4) dx$

e) $\int_{-\pi/2}^{\pi/2} x^2 \sin^2(x^3) \cos(x^3) dx$

f) $\int_2^3 \frac{x^2 + 1}{(x-1)^4} dx$

g) $\int_{-\pi/4}^{\pi/4} (\sin^5 x + x^2 \tan x) dx$