MATH 221 **NAME:**
CALCULUS II **ID#:**

TAKE-HOME EXAM 1

Solve the following problems showing all your work for full credit.

1. Find the antiderivative $F(x)+C$ for each of the following:

a) (2 pts.) $f(x)=3x^{\frac{2}{3}}$

b) (2 pts.) $f(x)=3x^2+\sqrt{x}$

c) (3 pts.) $f(x)=x^2\left(x^3+5x^2-3x+\sqrt{3}\right)$

d) (3 pts.) $f(x)=\dfrac{\sqrt{2x}+x^2}{x^4}$

2. Evaluate the integral:

a) (2 pts.) $\int\left(e^{3x}+4^x\right)dx$

b) (3 pts.) $\int\dfrac{s(s+1)^2}{\sqrt{s}}\,ds$

c) (2 pts.) $\int \left(t^2 - 2\cos t\right)dt$

d) (2 pts.) $\int \frac{x^2 + 5x + 6}{x + 3}dx$

e) (2 pts.) $\int \left(\pi x^3 + 1\right)^4 3\pi x^2 dx$

f) (3 pts.) $\int \left(5x^2 + 1\right)\sqrt{5x^3 + 3x - 2}dx$

g) (3 pts.) $\int \frac{3y}{\sqrt{2y^2 + 5}}dy$

3. (2 pts.) Given $\int_0^3 f(x)dx = 4$ and $\int_8^0 f(x)dx = -10$, find $\int_3^8 f(x)dx$.

4. Evaluate the definite integral:

a) (2 pts.) $\int_{1}^{3} \left(3x^2 + 5x - 4\right) dx$

b) (3 pts.) $\int_{1}^{4} \left(3 - |x - 3|\right) dx$

c) (2 pts.) $\int_{-\pi/2}^{\pi/2} \left(2t + \cos t\right) dt$

d) (2 pts.) $\int_{0}^{1} \cos(4x - 4) dx$

e) (3 pts.) $\int_{-\pi/2}^{\pi/2} x^2 \sin^2(x^3) \cos(x^3) dx$

f) (3 pts.) $\int_{2}^{3} \frac{x^2 + 1}{(x-1)^4} dx$

g) (2 pts.) $\int_{-\pi/4}^{\pi/4} (\sin^5 x + x^2 \tan x) dx$

5. (3 pts.) Find the area of the region bounded by the graphs of the equations $y = 1 + \sqrt[3]{x}$, $x = 0$, $x = 8$, $y = 0$.

6. Find the area of the region bounded by the graphs of the functions:

a) (3 pts.) $y = -\frac{3}{8}x(x-8)$, $y = 10 - \frac{1}{2}x$, $x = 2$, $x = 8$

b) (3 pts.) $y = \sqrt[3]{x}$, $y = x$

c) (3 pts.) $x = y^2 - 2y$, $x - y - 4 = 0$

d) (3 pts.) $x = 4y^4$, $x = 8 - 4y^4$

7. Set up and evaluate the integral that gives the volume of the solid generated by revolving the region bounded by the graphs of the functions $f(x) = \frac{1}{2}x^3$, $y = 4$, $x = 0$ about

a) (3 pts.) the x-axis;

b) (3 pts.) the y-axis;

c) (3 pts.) the line $y = 4$;

d) (3 pts.) the line $y = 6$;

e) (3 pts.) the line $x = 2$;

f) (3 pts.) the line $x = 3$.

8. Find the volume of the solid generated by revolving the region bounded by the graphs of the functions

a) (3 pts.) $y = 6x$ and $y = 6x^2$ about the x-axis;

b) (3 pts.) $xy = 6$, $y = 2$, $y = 6$ and $x = 6$ about the line $x = 6$;

c) (3 pts.) $y = x^2$ and $y = 4x - x^2$ about the line $x = 2$;

d) (3 pts.) $y = \sqrt{x}$, $y = 0$ and $x = 4$ about the line $x = 6$.

9. (3 pts.) Find the arc length of the graph of the function $y = \frac{x^4}{8} + \frac{1}{4x^2}$ over the interval $[1,2]$.

10. Set up and evaluate the definite integral for the area of the surface generated by revolving the curve

a) (3 pts.) $y = \frac{x^4}{8} + \frac{1}{4x^2}$, $1 \le x \le 3$ about the x-axis;

b) (3 pts.) $x = \sqrt{2y - 1}$, $\frac{5}{8} \le y \le 1$, about the y-axis.