

**MATH 221
CALCULUS II**

**NAME:
ID#:**

TAKE-HOME EXAM 3

Solve the following problems showing all your work for full credit.

1. (4 pts.) Write an expression for the n th term of the sequence

a) $\frac{3}{2}, \frac{4}{5}, \frac{5}{8}, \frac{6}{11}, \frac{1}{2}, \dots$

b) $-\frac{1}{3}, \frac{1}{2}, -\frac{3}{4}, \frac{9}{8}, -\frac{27}{16}, \dots$

2. (12 pts.) Determine the convergence or divergence of the sequence with the given n th term. If the sequence converges, find its limit.

a) $a_n = \frac{7^n}{6^n}$

b) $a_n = \frac{3^n}{8^n}$

c) $a_n = \frac{\sqrt[3]{n}}{\sqrt[3]{n} + 1}$

d) $a_n = \cos \frac{2}{n}$

e) $a_n = \frac{2n^2 + 1}{n + 1} - \frac{2n^2 + 1}{n - 1}$

f) $a_n = \frac{(n - 2)!}{n!}$

3. (33 pts.) Determine whether the series converges conditionally or absolutely, or diverges.

a) $\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$

b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n}$

c) $\sum_{n=1}^{\infty} \left(1 + \frac{3}{n}\right)^n$

d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$

e) $\sum_{n=1}^{\infty} \frac{5n-3}{n^2-2n+5}$

f) $\sum_{n=1}^{\infty} \frac{(-1)^n}{4n-1}$

g) $\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n^3}$

$$\text{h) } \sum_{n=2}^{\infty} \frac{(-1)^n \ln n^3}{n}$$

$$\text{i) } \sum_{n=1}^{\infty} \frac{(-1)^n n^n}{n!}$$

$$\text{j) } \sum_{n=1}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 5 \cdot 8 \cdots (3n-1)}$$

$$\text{k) } \sum_{n=1}^{\infty} \frac{(-1)^n 4^n}{n3^n}$$

4. (6 pts.) Find the sum of the convergent series.

$$\text{a) } \sum_{n=0}^{\infty} \frac{2^{n+2}}{3^n}$$

$$\text{b) } \sum_{n=0}^{\infty} \left(\frac{2^n}{3^n} - \frac{1}{(n+1)(n+2)} \right)$$

5. (24 pts.) Find the interval of convergence of the power series.

a)
$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(n+1)(n+2)}$$

b)
$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$$

c)
$$\sum_{n=0}^{\infty} \frac{(3x)^n}{(2n)!}$$

d)
$$\sum_{n=0}^{\infty} (-1)^{n+1} (n+1)x^n$$

e)
$$\sum_{n=1}^{\infty} \frac{n! x^n}{(2n)!}$$

f)
$$\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdots (2n)}{2 \cdot 5 \cdot 8 \cdots (3n-1)} x^n$$
 (find only the radius of convergence)

6. Find the terms through x^5 in the Maclaurin series for the function

a) (3 pts.) $f(x) = e^{-x}$

b) (3 pts.) $f(x) = \frac{x}{x+1}$

c) (3 pts.) $f(x) = x^2 \tan x$

d) (4 pts.) $f(x) = (1+x)^{3/2}$

7. (8 pts.) Find the Taylor series in $x-a$ through $(x-a)^3$.

a) $f(x) = \sqrt[3]{x}$, $a = 8$

b) $f(x) = \frac{2}{x^2}$, $a = 2$