

**MATH 221
CALCULUS II**

**NAME:
ID#:**

TAKE-HOME EXAM 4

Solve the following problems showing all your work for full credit.

1. (5 pts.) Find parametric equations and a parameter interval for the motion of a particle that starts at $(a,0)$ and traces the ellipse $\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) = 1$ twice counterclockwise.
2. (5 pts.) For the following parametric equation and parameter interval for the motion of a particle in the xy -plane identify the particle's path by finding a Cartesian equation for it: $x = 1 + \sin t$, $y = \cos t - 2$, $0 \leq t \leq \pi$.
3. (5 pts.) Find an equation for the line tangent to the curve $x = -\sqrt{t+1}$, $y = \sqrt{3t}$ at the point $t = 3$.
4. (5 pts.) Find the length of the curve $x = t^3$, $y = \frac{3t^2}{2}$, $0 \leq t \leq \sqrt{3}$.

5. (5 pts.) Find the area of the surface generated by revolving the curve $x = \frac{2}{3}t^{\frac{3}{2}}$, $y = 2\sqrt{t}$, $0 \leq t \leq \sqrt{3}$ about the y-axis.

6. (15 pts.) Replace the polar equations by equivalent Cartesian equations:

a) $r^2 \sin 2\theta = 2$

b) $r = 3 \cos \theta$

c) $r^2 = 4r \sin \theta$

7. (5 pts.) Sketch the graph of the polar equation $r = 3(1 - \cos \theta)$.

8. (5 pts.) Find the points of intersection of the graphs of the equations $r = 1 + \cos \theta$ and $r = 3 \cos \theta$.

9. (5 pts.) Find the length of the curve $r = 2a \cos \theta$ over the indicated interval $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

10. (5 pts.) Find the area inside one loop of the lemniscate $r^2 = 9 \sin 2\theta$.

11. (25 pts.) Find the vertices, foci, directrices, and sketch the graph of the curve

a) $y^2 + 4y + 8x - 12 = 0$

b) $16x^2 + 25y^2 - 64x + 150y + 288 = 0$

c) $9x^2 - 4y^2 + 54x + 8y + 78 = 0$

12. (15 pts.) Find a polar equation for the conic with its focus at the pole.

a) parabola, $e = 1$, directrix $y = 1$

b) ellipse, $e = \frac{3}{4}$, $y = -2$

c) hyperbola with vertices $\left(1, \frac{3\pi}{2}\right)$ and $\left(9, \frac{3\pi}{2}\right)$