# CCSU
## DEPARTMENT OF MATHEMATICAL SCIENCES
# COLLOQUIUM

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3:00 – 4:00 PM
Maria Sanford, Room 101

# CONSTRUCTIONS OF THE REGULAR N-GONS: COMPARING EUCLID'S *ELEMENTS* WITH GAUSS'S *DISQUISITIONES ARITHMETICAE*


## ROGER BILISOLY
### CENTRAL CONNECTICUT STATE UNIVERSITY



**Abstract:** Euclid's *Elements* was a groundbreaking mathematics book published circa 300 BC. Among other propositions, it famously shows how to construct the equilateral triangle, square, and the regular hexagon, pentagon, and 15-gon using only a compass and unmarked straightedge. However, it was not until 2100 years later that Carl Gauss discovered how to construct a regular 17-gon, which proved that Euclid's list was incomplete.

This talk discusses how Gauss's approach compares to the classical constructions. It turns out that at age 19, in 1796, he used complex numbers, cyclotomic polynomials, quadratic field extensions, abelian Galois groups, and cosets to construct regular n-gons. This is especially impressive because this was long before the development of abstract algebra in the second half of the 19th century.

The case of the regular pentagon is discussed in detail before turning to Gauss's breakthrough discovery that the regular 17-gon was constructable. Below is a facsimile from section 365 of the *Disquisitiones arithmeticae*, which gives an exact expression for cos(2π/17). How did Gauss derive this? Come and find out.

$$-\tfrac{1}{16}+\tfrac{1}{16}\sqrt{17}+\tfrac{1}{16}\sqrt{(34-2\sqrt{17})}-\tfrac{1}{8}\sqrt{(17+3\sqrt{17}-\sqrt{(34-2\sqrt{17})}-2\sqrt{(34+2\sqrt{17})})}$$


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For further information: gotchevi@ccsu.edu; 860-832-2839; http://mathcolloquium.sites.ccsu.edu/